KVPY → SB

Past Year Papers (Solved)

And

Practise Questions

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24/06/2017

Dedicated

to all my students

who

in some way have been

my teachers

CONTENTS

How to use this handout.

- First solve all the previous year papers. You will get a hang of the concept, approach and topics.
- Then take 20 questions from Practise set & allocate 45 minutes. Check your score. Revise those unattempted questions.
- In such way you can give 9 exams and keep a track on your score.
- Try to finish year papers by July & August. Then take another 2 months for the Practise set. So By November beginning you will be prepared.

2015 Part-1

1) Two distinct polynomial $f(x)$ and $g(x)$ are defined as follows, $f(x) = x^2 + ax + 2$, $g(x) = x^2 + 2x + a$.
If the Equation $f(x) = 0$, $g(x) = 0$ have a common root, then the sum of roots of the eqn. $f(x) + g(x) = 0$ is

(a) $-1/2$ (b) 0 ✓(c) $1/2$ (d) 1

Sol: Let α be common root.

$$\left.\begin{aligned} f(\alpha) &= \alpha^2 + a\alpha + 2 = 0 \\ g(\alpha) &= \alpha^2 + 2\alpha + a = 0 \end{aligned}\right] \quad \frac{\alpha^2}{a^2-4} = \frac{-2}{a-2} = \frac{1}{2-a}$$

$$\therefore \alpha = \frac{(2-a)}{2-a} \Rightarrow \alpha = 1.$$

If $\alpha = 1$, then $a - 2 = -(a^2-4) \Rightarrow (a-2) = -(a-2)(a+2)$

$(a-2)(a+3) = 0 \Rightarrow a = 2, -3.$

Now $f(x) + g(x) = 2x^2 + (a+2)x + a + 2$

$\therefore$ Sum of roots $= -\frac{a+2}{2}$; If $a = -3$, then sum $= \frac{1}{2}$.

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2) If m is the smallest natural no. then $m + 2m + 3m + \dots + 99m$ is a perfect square, then the no. of digits in m^2 is

a) 1 b) 2 ✓c) 3 d) more than 3

Sol: $m(1 + 2 + \dots + 99) = m \times \frac{99 \times 100}{2} = 9 \times 25 \times 22 \times m$ is a perfect square, then $m = 22$.

$\therefore$ no. of digits in $m^2 = 3$

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3) Let, x, y, z be positive reals. Which of the following implies $x = y = z$?

i) $x^3 + y^3 + z^3 = 3xyz$
ii) $x^3 + y^2z + yz^2 = 3xyz$
iii) $x^3 + y^2z + z^2x = 3xyz$
iv) $(x+y+z)^3 = 27xyz$

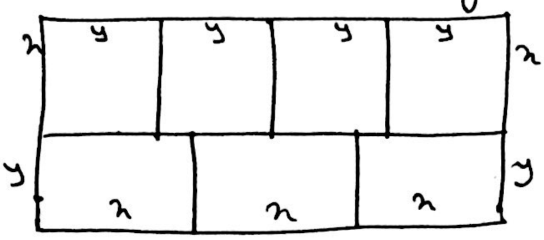
(a) I, IV only
✓(b) I, II, IV only
(c) I, II, III only
(d) All of them.

Sol: $x^3 + y^3 + z^3 = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz.$

Sof $x = y = z$, $\therefore$ $x^3 + y^3 + z^3 = 3xyz$. ---- (i)

$x^3 + y^2z + yz^2 = 3xyz$ --- (ii)

4) In the figure given below, a rectangle of perimeter 76 units is divided into 7 congruent rectangles



What is the perimeter of each of the smaller rectangles?

(a) 38 (b) 32 ✓(c) 28 (d) 19.

Sol: Let sides of rectangle be x & y.

then $5x + 6y = 76$

Also $4y = 3x$ $\Rightarrow$ $y = 6,\ x = 8$, so Perimeter $= 2(x+y)$ $= 28$.

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5) The largest non negative integer K such that 24^K divides $13!$ is

(a) 2 ✓(b) 3 (c) 4 (d) 5

Sol: $24^K = (2^3 \times 3)^K$

Exponent of 2 in $13! = \left[\frac{13}{2}\right] + \left[\frac{13}{2^2}\right] + \left[\frac{13}{2^3}\right] = 10$

" " 3 in $13! = \left[\frac{13}{3}\right] + \left[\frac{13}{3^2}\right] = 5$

$\therefore (2^3 \times 3)^3$ will divide $13!$ $\therefore K = 3$.

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6) In $\triangle ABC$, Points X and Y are on AB and AC respectively, such that XY is $\parallel$ BC. Which of the two following always hold? (here [PQR] denotes area of $\triangle PQR$)

i) $[BCX] = [BCY]$

ii) $[ACX] \cdot [ABY] = [AXY][ABC]$

a) Neither (i) nor (ii) b) (i) only c) (ii) only ✓(d) both (i) and (ii).

Sol:

$[BCX] = [BCY]$ same base, same ht.

Let A be $\vec{0}$, $\vec{AB}$ be $\vec{b}$, $\vec{AC}$ be $\vec{c}$ $\therefore \vec{AX} = \lambda \vec{b}$, $\vec{AY} = \lambda \vec{c}$

$\therefore [ACX] = \frac{1}{2}\lambda|\vec{b} \times \vec{c}|$; $\triangle ABC = \frac{1}{2}|\vec{b} \times \vec{c}|$

$[ABY] = \frac{1}{2}\lambda|\vec{b} \times \vec{c}|$;

$[AXY] = \frac{1}{2}\lambda^2|\vec{b} \times \vec{c}|$;

so, (i) & (ii) both are correct.

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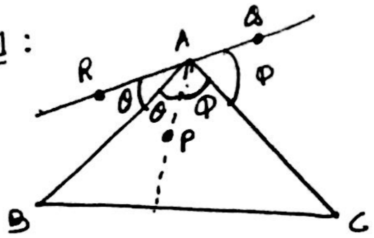
7) Let P be an interior point of $\triangle ABC$. Let Q and R be the reflections of P in AB, AC, respectively. If Q, A, R are collinear, then $\angle A =$ (A) 30° (B) 60° (C) 90° (d) 120°.

Sol:



$2\theta + 2\phi = 180^\circ$
$\theta + \phi = 90^\circ$

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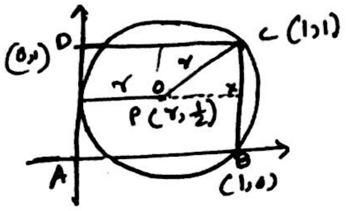
8) Let ABCD be a square of side length 1, and 'S' a circle passes through B and C, and touches AD. Radius of 'S' is (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ ✓(d) $\frac{5}{8}$.

Sol:



$PC = r$

$(1-r)^2 + (1-\frac{1}{2})^2 = r^2$

$\Rightarrow 1 - 2r + \frac{1}{4} = 0$

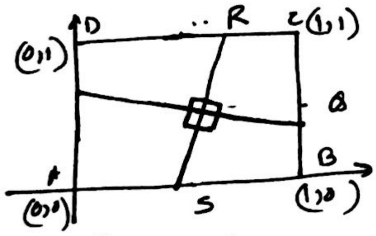
$r = 5/8$

---

9) Let ABCD be a square of side length 1. Let P, Q, R, S be points in the interiors of the sides AD, BC, AB, CD, respectively, such that PQ and RS intersect at 90°. If $PQ = \frac{3\sqrt{3}}{4}$, then RS =

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{3\sqrt{3}}{4}$ (c) $\frac{\sqrt{2}+1}{2}$ (d) $4 - 2\sqrt{2}$

Sol:



P(0, a)
Q(1, c)
R(d, 1)
S(b, 0)

$m_{PQ} \times m_{RS} = -1$

$(c-a) \times \frac{(0-1)}{b-d} = -1$

$\Rightarrow -(c-a) = d - b$

$\Rightarrow a - c = d - b.$

$PQ^2 = \frac{27}{16} = 1 + (c-a)^2$

$RS = \sqrt{(b-d)^2+1} = \sqrt{(a-c)^2+1} \Rightarrow RS^2 = (a-c)^2 + 1 \Rightarrow (a-c)^2 = RS^2 - 1$

$\therefore \frac{27}{16} = \cancel{1} + RS^2 - \cancel{1} \Rightarrow RS = \frac{3\sqrt{3}}{4}.$

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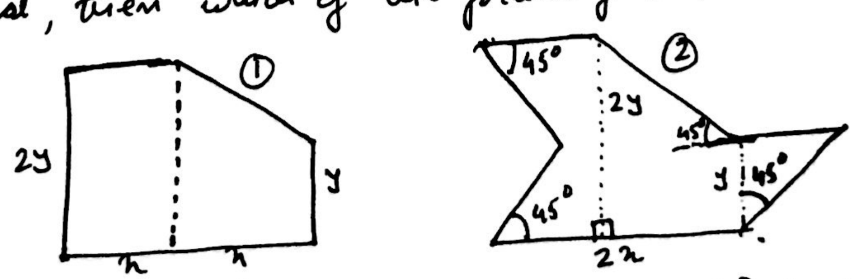
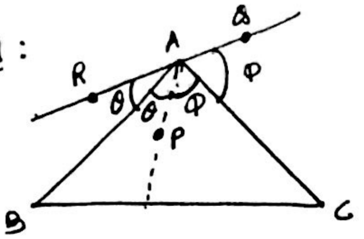
10) In the figure given below, if the areas of two regions are equal, then which of the following is true?

Sol:



(a) $x = y$ (b)✓ $x = 2y$ (c) $2x = y$ (d) $x = 3y$

Sol:



$$2\theta + 2\varphi = 180^\circ$$
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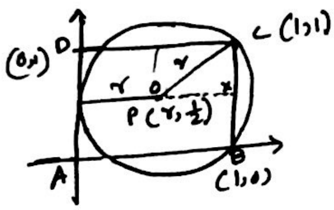

$PC = r$

$(1-r)^2 + (1-\frac{1}{2})^2 = r^2$

$\Rightarrow 1 - 2r + \frac{1}{4} = 0$

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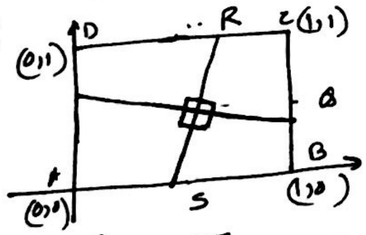
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{3\sqrt{3}}{4}$ (c) $\frac{\sqrt{2}+1}{2}$ (d) $4 - 2\sqrt{2}$

Sol:



$P(0,a)$

$Q(1,c)$

$R(d,1)$

$S(b,0)$

$m_{PQ} \times m_{RS} = -1$

$(c-a) \times \frac{(0-1)}{b-d} = -1$

$\Rightarrow -(c-a) = d - b$

$\Rightarrow a - c = d - b.$

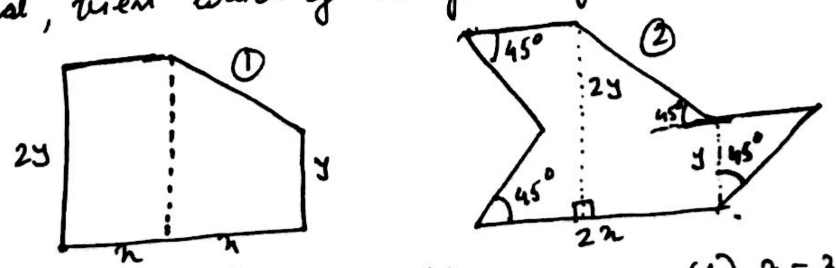
$PQ^2 = \frac{27}{16} = 1 + (c-a)^2$

$RS = \sqrt{(b-d)^2+1} = \sqrt{(a-c)^2+1} \Rightarrow RS^2 = (a-c)^2 + 1 \Rightarrow (a-c)^2 = RS^2 - 1$

$\therefore \frac{27}{16} = \cancel{1} + RS^2 - \cancel{1} \Rightarrow RS = \frac{3\sqrt{3}}{4}.$

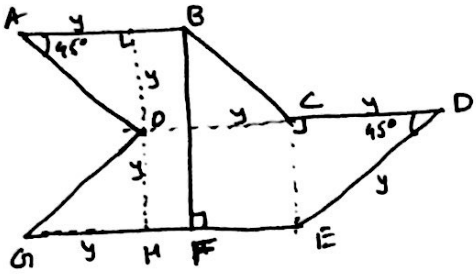
10) In the figure given below, if the areas of two regions are equal, then which of the following is true?

Sol:



(a) $x = y$ ✓(b) $x = 2y$ (c) $2x = y$ (d) $x = 3y$

Area of figure (1) $A_1 = x\times 2y + \frac{1}{2}(y+2y)x = \frac{7}{2}xy$.
" " figure (2) $A_2 = (\frac{1}{2}\times y\times y)\times 4 + (2x-2y)\,2y + y^2$



($HF = 2x-2y$)

$\therefore A_1 = A_2$

$\frac{7}{2}xy = 2y^2 + 4xy - 4y^2 + y^2$

$\frac{7}{2}xy = 4xy - y^2 \Rightarrow x = 2y$.

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11) A man standing on a railway platform noticed that a train took 21 s to cross the platform (this means the time elapsed from the moment the engine enters the platform till the last compartment leaves the platform) which is 88 mts. long, and that it took 9s to pass him. Assuming that the train was moving with uniform speed, what is the length of the train in mts?

(a) 55 (b) 60 ✓(c) 66 (d) 72.

Sol: Let speed be x.

$9x + 88 = 21x \Rightarrow x = \frac{88}{12}$

Length $= \frac{88}{12}\times 9 = 66$

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12) The least positive integer n for which $\sqrt[3]{n+1} - \sqrt[3]{n} < \frac{1}{12}$ is

(a) 6 (b) 7 ✓(c) 8 (d) 9

Sol: $(n+1)^{1/3} < n^{1/3} + \frac{1}{12}$

Cubing both sides: $n+1 < n + \frac{1}{1728} + 3n^{2/3}\cdot\frac{1}{12} + 3n^{1/3}\cdot\frac{1}{144}$

$\Rightarrow \frac{1727}{1728} < \frac{3}{12}n^{1/3}\left(n^{1/3} + \frac{1}{12}\right) \Rightarrow \frac{1727}{432} < n^{1/3}\left(n^{1/3}+\frac{1}{12}\right)$

$\Rightarrow 3.9 < (2^3)^{1/3}\left(2^{3\times\frac{1}{3}} + \frac{1}{12}\right)$ (By option testing)

so $n = 8$

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13) Let $n>1$ be an integer. Which of the following sets of numbers necessarily contains a multiple of 3?

(a) $n^{19}-1, n^{19}+1$ ✓(b) $n^{19}, n^{38}-1$ (c) $n^{38}, n^{38}+1$ (d) $n^{38}, n^{19}-1$.

Sol: Let n be multiple of 3 $\Rightarrow n^{19} \equiv 0 \pmod 3$

By Fermat's theorem, If a, P are co prime then $a^{P-1} \equiv 1 \pmod P$ $\therefore n^2 \equiv 1 \pmod 3$

Now $n^{38} \equiv 1 \pmod 3 \Rightarrow n^{38} - 1 \equiv 0 \pmod 3$

so option (b)

By Binomial theorem process would have been lengthy. So I solve by Congruence of nos.

14) The no. of distinct primes dividing $12! + 13! + 14!$ is
✓(a) 5 (b) 6 (c) 7 (d) 8

Sol: $12! + 13! + 14! = 12!(1+13+14\times13)$
$= 12!(196)$ this is divisible by 2, 3, 5, 7, 11,

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15) How many ways are there to arrange the letters of the word EDUCATION so that all the following 3 conditions hold:
- The vowels occur in the same order (EUAIO)
- consonants occur in same order (DCTN)
- no two consonants are next to each other.

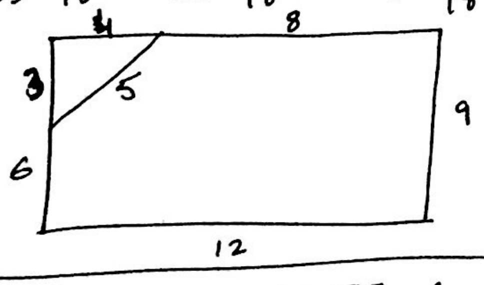
✓(a) 15 (b) 24 (c) 72 (d) 120

Sol: _ E _ U _ A _ I _ O _ (1 way)
'_' Place for consonants: $^6C_4 = 15$

PART – II

16) A Triangular corner is cut from a rectangular piece of paper and the resulting pentagon has sides 5, 6, 8, 9, 12 in some order. The ratio of the area of the rectangle to that of pentagon is
(a) $\frac{11}{18}$ (b) $\frac{13}{18}$ (c) $\frac{15}{18}$ ✓(d) $\frac{18}{17}$

Sol:



$\frac{\text{Rectangle}}{\text{Pentagon}} = \frac{12\times9}{12\times9 - \frac{1}{2}\times3\times4}$
$= \frac{18}{17}$.

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17) for real no. x, let $[x]$ denote the largest integer less than or equal to x, and let $\{x\} = x-[x]$. The no. of solution of x to the equation $[x]\{x\} = 5$ with $0 \le x \le 2015$ is
(a) 0 (b) 3 (c) 2008 ✓(d) 2009.

Sol: $\{x\} = x - [x]$
given $[x]\{x\} = 5 \Rightarrow$ Integer × Fraction = Integer.
Possible solutions are $(6\times\frac{5}{6}), (7\times\frac{5}{7}), \ldots\ldots (2015\times\frac{5}{2015})$
so $x = 6, 7, \ldots 2015 \Rightarrow$ No. of sol is 2009.
